Measuring Chaos in a Double Pendulum

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Motivation & Background

- Simple system, chaotic motion

- Chaos: small $\Delta_{\text{initial}} \rightarrow$ large $\Delta_{\text{final}}$

- Exponential separation characterized by Lyapunov Exponent
Two Modes of Attack

Simulation
- MATLAB generated
- Runge-Kutta Method
- No Friction

Physical Pendulum
- Double-bar pendulum
- Released from high angle
- Circular dots for tracking
- Casio EX-F1 at ~5 feet
- 600 fps analyzed in MATLAB
Circle Detection & Tracking

- Circle Detection – Circular Hough
- Accumulation Array
- Circle Differentiation
- Angles Extracted
Phase Space Plots

- 4 parameters: $\theta, \Phi, \delta\theta, \delta\Phi$
- Angle 1 vs. its Angular Velocity
- Chaotic $\rightarrow$ Periodic
Lyapunov Exponent

- 4 parameters: $\theta, \Phi, \delta\theta, \delta\Phi$
- $|\delta Z(t)| \approx |\delta Z(t_0)| e^{\lambda t}$
- At least 1 positive to show chaotic behavior

- Rosenstein Method
  - Nearest Neighbors
  - Temporal Separation
Challenges/Problems

- Camera
  - Lighting
- Energy Function
- Tracking & Interpolation
  - Circle Sizes
  - Could Switch Direction
Future Analysis

- Fractal Dimension of attractor
- Angle 2 vs. its velocity
- Use Box-counting method
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Questions?
Lyapunov Exponent Calculation
Attractor Plot