Dispersion in Optical Fibre (I)

Dispersion limits available bandwidth
As bit rates are increasing, dispersion is becoming a critical aspect of most systems
Dispersion can be reduced by fibre design
Optical source selection is important

Dispersion in an Optical Fibre

- Multimode fibre (LAN systems)
- Modal dispersion
- Chromatic dispersion
- Polarization Mode dispersion (PMD)

Singlemode fibre (Telecom systems)
Why is dispersion a problem?

- Dispersion → Pulse spreading → Intersymbol Interference → Degrades error rate

Example

Data at fibre input

Optical Fibre

Data at fibre output

Pulse spreading makes it more difficult to distinguish "0"

Dispersion and Bit Rate

- The higher the dispersion the longer the bit interval which must be used
- A longer bit interval means fewer bits can be transmitted per unit of time
- A longer bit interval means a lower bit rate

Conclusion: The higher the dispersion the lower the bit rate

Dispersion Example

Photo of Input and Output pulses for a 200 micron core Polymer Clad Silica fibre showing pulse broadening (dispersion)

Modal Dispersion

- In a multimode fibre different modes travel at different velocities
- If a pulse is constituted from different modes then intermodal dispersion occurs
- Modal dispersion is greatest in multimode step index fibres
- The drive to reduce modal dispersion led to the development of graded index multimode fibre and singlemode fibre.
- A ray model can give an adequate description of modal dispersion
Modal Dispersion

- Modal dispersion is greatest in multimode step index fibres.
- The more modes the greater the modal dispersion.
- Typical bandwidth of a step index fibre may be as low as 10 MHz over 1 km.

Analysis for Modal Dispersion

Estimating Modal Dispersion (Step Index Fibre)

- Assume:
  - Step index fibre
  - An impulse-like fibre input pulse
  - Energy is equally distributed between rays with paths lying between the axial and the extreme meridional.
- What is the difference in delay for the two extremes over a linear path length L?

Step Index Modal Dispersion: Analysis (I)

Transmission distance = L

\[ T_{\text{max}} = \text{Transmission time for extreme meridional ray} \]

\[ T_{\text{min}} = \text{Transmission time for axial ray} \]

\[ \delta t = T_{\text{max}} - T_{\text{min}} \]
**Step Index Modal Dispersion: Analysis (II)**

\[
T_{\text{min}} = \frac{\text{Distance}}{\text{Velocity}} = \frac{L}{c/n_1} = \frac{L n_1}{c}
\]

To find \( T_{\text{max}} \) realise that the ray travels a distance \( h \) but only travels a distance \( d \) toward the fibre end (\( d < h \)). So if the fibre length is \( L \) then the actual distance travelled is:

\[
\frac{h L}{d}
\]

**Step Index Modal Dispersion: Analysis (III)**

Using Snell's law:

\[
T_{\text{max}} = \frac{L n_1}{c n_2} \cos \theta
\]

Using simple trigonometry

\[
\sin \theta = \frac{n_2}{n_1} = \cos \theta
\]

\[
T_{\text{max}} = \frac{L n_1^2}{c n_2}
\]

Delay difference \( \delta t = T_{\text{max}} - T_{\text{min}} = \frac{L n_1^2}{c n_2} - \frac{L n_1}{c}
\]

**Step Index Modal Dispersion: Analysis (IV)**

\[
\delta t = \frac{L n_1^2}{c n_2} \left( \frac{n_1 - n_2}{n_1} \right) = \frac{L \Delta n_1^2}{c n_2} \quad \text{Assumes } \Delta \ll 1
\]

Show for yourselves that:

\[
\delta t = \frac{L (NA)^2}{2 c n_1}
\]

**Impulse Response for Step Index Fibre**

- Assume an impulse input to the fibre
- Output is a pulse of uniform amplitude over a time period \( T_{\text{max}} - T_{\text{min}} = \delta t \)
- Output pulse of width \( \delta t \) is thus the impulse response of the fibre.
- Assuming an output pulse amplitude of \( 1/\delta t \), the impulse response \( h(t) \) is given by:

\[
h(t) = \frac{1}{\delta t} \quad -\delta t/2 < t < +\delta t/2
\]

\[
h(t) = 0 \quad \text{elsewhere}
\]
Transfer Function for a Step Index Fibre

• Take the Fourier transform of the impulse response
• The transfer function of the fibre $H(f)$ is given by:

$$H(f) = \text{sinc} f \delta_t$$

Note: $\text{sinc} x = \frac{\sin \pi x}{\pi x}$

Plot of $H(f)$ is sinc like
• First zero is at $1/\delta_t$, the so-called essential bandwidth for a system, $BW$

Bandwidth for a Step Index Fibre (I)

• Essential bandwidth, $BW$, for the fibre is $1/\delta_t$
• Based on the previous analysis $BW$ can be written as:

$$BW = \frac{2 c n_1}{L(NA)^2}$$

• $BW$ get smaller as fibre length $L$ increases
• High NA fibres have lower bandwidths, e.g., plastic fibre has high NA: Poor bandwidth
• Lowering NA to improve bandwidth makes source coupling more difficult as the acceptance angle decreases

Bandwidth Problem: Plastic Optical Fibre

• Conventional plastic optical fibre is step index, low bandwidth
• NA is about 0.4, core refractive index is about 1.5
• Show that the $BW$ over 1 km is about 6 MHz
• Measured values are about 6 to 10 MHz so analysis is about right

Reducing Modal Dispersion
Reducing Modal Dispersion

Reduce the difference between the propagation velocities of different modes

- Graded index fibre design

Reduce the number of modes to one

- Single mode fibre design

The Profile Parameter and Intermodal Dispersion

- Recall that the profile parameter $\alpha$ for a graded index fibre dictates the shape of the refractive index profile
- Why does the profile parameter $\alpha$ used for graded index fibre has a common value of about 2?
- It can be shown that the optimum value of $\alpha$ that maximises the bandwidth of GI fibre is given by:

$$\alpha = 2.(1-\Delta)$$

- A common $\Delta$ value for GI multimode fibre is 0.02 (2%) (Lucent 62.5/125 µm)
- For this $\Delta$ value the optimum profile parameter $\alpha$ has a value of 1.96.

Reducing Dispersion using a Graded Index Fibre

Light ray (a) and (b) are refracted progressively within the fibre. Notice that light ray (a) follows a longer path within the fibre than light ray (b)

- Ray (a) follows a longer path, but the much of the path lies within the lower refractive index part of the fibre.
- Ray (b) follows a shorter path, but near the fibre axis where the refractive index is higher
- Since the velocity increases as the refractive index decreases the time delay between (a) and (b) is equalised

Variation in Modal Dispersion with the Profile Parameter

- Plot below shows variation in intermodal dispersion with the profile parameter.
- Plot assumes a $\Delta$ value of 1% for the fibre.
- Large value of $\alpha > 3$ means a profile approaching step index.
- Dispersion drops by more than 100:1 with $\alpha \approx 2$ by comparison with $\alpha > 3$
- Thus bandwidth of graded index is > 100 times higher than step index
Quantifying Dispersion in a GI Fibre (I)

• Very involved analysis
• As in the step index case one determines maximum time difference between the two most extreme modes
• Most common expression is:

\[ \delta t_{GI} = \frac{L \Delta n_1}{c.8} \]

• By comparison the equivalent value for a step index fibre has been shown to be:

\[ \delta t_{SI} = \frac{L \Delta n_1}{c n_2} \]

• Because of the \( \Delta^2 \) dependence for graded index the dispersion is much lower since \( \Delta \) is \( \ll 1 \).

Quantifying Dispersion in a GI Fibre (II)

• Using the formulas below and assuming an \( n_1 \) value of 1.5, plot the maximum time delay or dispersion for a step index and a graded index fibre for values of \( \Delta \) from 0.01 to 0.05 using the units “ns per km” and using a common axis for \( \Delta \).

\[ \delta t_{GI} = \frac{L \Delta n_1}{c.8} \]

\[ \delta t_{SI} = \frac{L \Delta n_1}{c n_2} \]

Using Singlemode Optical Fibre to Eliminate Modal Dispersion

• No modal dispersion since only one mode propagates
• Most effective way to overcome modal dispersion
• Potential bandwidth is in the order of 20 THz